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DECISION, IRREVERSIBILITY AND FLEXIBILITY: THE IRREVERSIBILITY EFFECT RE-EXAMINED

ABSTRACT. In a seminal article C. Henry (1974) presented the irreversibility effect, which pointed out that under uncertainty, the optimal sequence of decisions depends on not only the payoffs, but also the flexibility, in terms of availability of future options, associated with each decision. But the irreversibility effect pertained to certain particular conditions and definitions. In this paper, a more general model is developed to re-examine the notion of an irreversible decision, its relation with flexibility and the irreversibility effect. It is shown through two propositions that the irreversibility effect need not hold always and the notion of irreversibiity can be used only under certain circumstances to derive the optimal sequence of decisions ex-ante.

Keywords: irreversibility, flexibility, irreversibility effect, information, gains from information.

1. INTRODUCTION

A decision takes on the characteristics of irreversibility to the extent that it shrinks the space of available options. In other words, an irreversible decision is one which, if taken, results in not being able to exercise (for a long time or forever) some option that was available earlier. In two articles, Arrow and Fisher (1974) and Henry (1974a) independently studied the link between irreversibility, uncertainty and information. Furthermore Henry stated the link between irreversibility, uncertainty and information explicitly in a proposition called the 'irreversibility effect'. This stated that an irreversible decision that yields better payoffs as compared to a reversible decision under a particular situation, may with more information (and under the same situation) yield lower payoffs than the reversible decision. 'More information' here connotes an increased capability to anticipate with greater accuracy and precision, the state of the world tomorrow. Through different analyses they arrived at the same normative conclusion: in the face of anticipated increases in information, it might be better to take a reversible rather than an irreversible decision.

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This result inspired two streams of literature. One studied the relation between decision making under exogeneous uncertainty (given by geometric Brownian motion) with irreversibility. A good survey may be found in Pindyck (1991). The second stream studied further refinements of the irreversibility effect (Freixas and Laffont, 1984, 1986) and its applications to various situations (Freixas, 1987). Cohendet and Llerena (1989) also present in their book an extensive survey on illustration and application of the irreversibility effect to investment, production and finance theories.

It is to be noted, however, that the irreversibility effect related to certain particular situations and irreversibility was always taken to be synonymous with loss of flexibility. In this connection, the objective of this paper is to examine what happens to the irreversibility effect in a general context.

This paper is organized as follows. Firstly the relation between flexibility and irreversibility and the models of the irreversibility effect are examined. Then two propositions on the robustness of the irreversibility effect are presented.

2. A TWO PERIOD MODEL OF DECISION MAKING

Before presenting the model we introduce the main notation used:

i, j, k, l:	general indices.
$\mathcal{A} = \{a_i; i = 1, 2, \ldots, I\}$:	set of first stage (intermediate) states.
$\mathcal{B} = [b_j; j = 1, 2, \ldots, J\}$:	set of second stage (terminal) states.
d_t:	decision taken at time t.
r_t:	reversible decision.
ir_t:	irreversible decision.
$d^*_{t+1}(d_t)$:	best response at time $t+1$ to decision d_t.
$D_t(o)$:	set of decisions available at initial state o.
$D_{t+1}(a_i \mid d_t)$:	set of decisions available at first stage state a_i when decision d_t is taken at the initial state o.
$P^o_{a_i}(d_t)$:	probability of reaching a_i from state o using decision d_t.

$U(d_t)$:	utility of intermediate reward at time $t+1$ associated with decision d_t.
$\mathcal{U}'(b_j)$:	utility of reward at time $t+2$ associated with terminal state b_j.

Let us consider a two period decision making problem as follows. Suppose at an initial state o a set of decisions $D_t(o)$ is available at time t for some decision maker. Every decision d_t in $D_t(o)$ can reach each of the first stage (intermediate) states a_i, $i = 1, 2, \ldots, I$ in $\mathcal{A}$ with probability $P^o_{a_i}(d_t)$. At every state a_i in $\mathcal{A}$, a set of decisions $D_{t+1}(a_i \mid d_t)$ is available at time $t+1$ as a consequence of having taken decision d_t at time t. Every decision d_{t+1} in $D_{t+1}(a_i \mid d_t)$ can then reach each of the second stage (terminal) states b_j, $j = 1, 2, \ldots, J$ in $\mathcal{B}$ with probability $P^{a_i}_{b_j}(d_{t+1})$. If decision d_{t+1} is not available at first stage state a_i after having taken d_t, $P^{a_i}_{b_j}(d_{t+1}) = 0$ for all states b_j in $\mathcal{B}$.

The problem of the decision maker is to plan *ex-ante* for a decision d_t and a set of state contingent decisions $d_{t+1} \in D_{t+1}(a_i \mid d_t)$ so as to maximize total utility obtained. But planning in time period t for state contingent decisions for time period $t+1$ will depend on the ability of the decision maker to recognize the intermediate states at time $t+1$ on arrival. For example, suppose the decision maker has to decide today whether to take an umbrella or a sun hat for tomorrow. If his house has a window, he could make state contingent plans such as: I will look out of my window and if it is raining I will take the umbrella; otherwise the hat. Whereas if he lived in a house with no windows (and he was not allowed to open the door to check the weather!) then he has no way to recognize the weather tomorrow and therefore cannot make state contingent plans beforehand. Thus planning of the decision maker at time t will depend crucially on his perception of his ability to distinguish intermediate states on arrival at time $t+1$. This leads to the following definitions.

2.1. *Information Set*

An information set is a union of first stage states among which the agent cannot distinguish.

2.2. Information Structure

An information structure S_θ is a union of information sets, say $s_{\theta,k}$, $k = 1, 2, \ldots, K$, which constitute a partition of the first stage states $\mathscr{A}$.

Complete information refers to the situation where no two first stage states are in the same information set. Incomplete information means that all first stage states are in the same information set. Finally, partial information refers to an information structure that is neither complete nor incomplete.

2.3. Information Refinement

An information structure $S_{\hat{\theta}}$ is said to be a refinement of information structure S_θ and yielding more information, if:

(i) for every information set $s_{\theta,j}$ in S_θ there is a set of information sets in $S_{\hat{\theta}}$ that constitute a partition of $s_{\theta,j}$.
(ii) for at least one information set s_{θ,k^*} in S_θ the partition in $S_{\hat{\theta}}$ consists of more than one set.

In other words, information refers to the ability to recognize first stage states on arrival and is said to be growing when the information structure becomes more refined.

At time t then, a decision maker has to choose a d_t available at initial state o and a K vector of decisions d_{t+1} corresponding to the K information sets $s_{\theta,k}$, $k = 1, 2, \ldots, K$ in his information structure S_θ.

There are two types of rewards; an intermediate reward associated with the initial decision taken d_t and a final reward associated with the sequence of decisions taken d_t, d_{t+1}. The intermediate reward is referred to as $U(d_t)$ and is obtained at time $t + 1$. It is further assumed that any two sequences of decisions which arrive at the same terminal state will obtain the same final reward. Therefore the final rewards obtained at time period $t + 2$ are associated with terminal states and given by the vector $\mathscr{U}'(b_j)$, $j = 1, 2, \ldots, J$. It is assumed that the preferences of the decision maker are such that his objective is to maximize the sum of his certain intermediate reward ($U(d_t)$) at time $t+1$ and the expected final reward at time $t+2$ ($V(d_t, d_{t+1}, S_\theta)$ under an information structure S_θ.

If a decision d_{t+1} is taken at first stage state a_i, the utility payoff in the final period is given according to the Von Neumann–Morgenstern expected utility function and is equal to:

$$\sum_{b_j \in \mathcal{B}} P_{b_j}^{a_i}(d_{t+1}) \cdot \mathcal{U}'(b_j)\,.$$

We know that when information is not complete a vector of decisional choices is made for time $t+1$, one for each information set. Now if decision d_t is chosen in time period t and the decision d_{t+1} is chosen at time $t+1$, the utility payoff in time period $t+2$ from each information set $s_{\theta,k}$ of information structure S_θ is given by:

$$v(d_t, d_{t+1}, s_{\theta,k}) = \sum_{a_i \in s_{\theta,k}} P_{a_i}^{o}(d_t) \cdot \left[\sum_{b_j \in \mathcal{B}} P_{b_j}^{a_i}(d_{t+1}) \cdot \mathcal{U}'(b_j) \right].$$

Finally the utility payoff in time period $t+2$ from the decisions d_t, d_{t+1} under an information structure S_θ, where S_θ is the union of the information sets $s_{\theta,k}$, $k = 1, 2, \ldots, K$, is:

$$V(d_t, d_{t+1}, S_\theta) = \sum_{k=1,2,\ldots,K} v(d_t, d_{t+1}, s_{\theta,k})\,.$$

Let $d_{t+1}^*(d_t)$ represent the K vector of decisions taken at time $t+1$ corresponding to the information sets $s_{\theta,k}$, $k = 1, 2, \ldots, K$, which maximizes $V(d_t, d_{t+1}, S_\theta)$. Then the problem of the decision maker is to maximize the sum of the intermediate and final utility payoffs $\pi(d_t, d_{t+1}, S_\theta)$ as shown below:

$$\max_{\{d_t \in D_t(o)\}} \pi(d_t, d_{t+1}, S_\theta) = U(d_t) + V(d_t, d_{t+1}^*(d_t), S_\theta)\,.$$

This framework of decisions is illustrated in Figure 1.

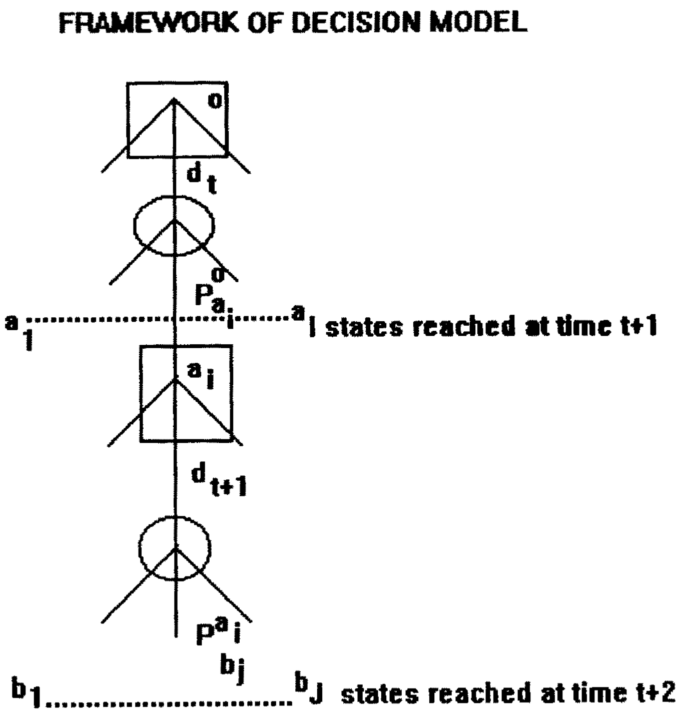

Fig. 1.

3. IRREVERSIBILITY AND FLEXIBILITY

We now present some definitions in which the symbol $\subset$ denotes 'proper subset of' and the symbol $\subseteq$ denotes 'subset of'.

3.1. *Loss of Flexibility*

(i) A decision d_t taken at the initial state o is said to reduce flexibility *over time* if at time $t+1$:

$$D_{t+1}(a_i \mid d_t) \subseteq D_t(o) \quad \text{for all states } a_i, i = 1, 2, \ldots, I$$

and $D_{t+1}(a_{i^*} \mid d_t) \subset D_t(o)$ for at least one state a_{i^*}.

(ii) A decision d_t reduces flexibility with respect to another decision $\bar{d}_t$ if:

$$D_{t+1}(a_i \mid d_t) \subseteq D_{t+1}(a_i \mid \bar{d}_t) \quad \text{for all states } a_i$$

$$D_{t+1}(a_{i^*} \mid d_t) \subset D_{t+1}(a_{i^*} \mid \bar{d}_t) \quad \text{for at least state } a_{i^*} \, .$$

Now suppose a reversible decision r_t and an irreversible decision ir_t were available at the initial state o. We would distinguish between them as follows.

3.2. Reversible Decision

A decision r_t taken at the initial state o is said to be reversible if at all first stage states a_i *reached* by the reversible decision we have:

$$D_{t+1}(a_i \mid r_t) \supseteq D_t(o) \, .$$

3.3. Irreversible Decision

A decision ir_t taken at the initial state o is said to be irreversible if at time $t+1$:

$$\{D_{t+1}(a_{i^*} \mid ir_t) \cap D_t(o)\} \subset D_t(o) \quad \text{for at least one state } a_{i^*} \, ;$$

and

$$D_{t+1}(a_i \mid ir_t) \subseteq D_{t+1}(a_i \mid r_t) \quad \text{for all states } a_i$$

which can be *reached* by the *reversible decision* r_t.

Note that the above definitions relate to loss of flexibility along two dimensions: firstly over time with respect to the original availability of decisions ($D_t(o)$) and secondly at time $t+1$ with respect to other decisions. A reversible decision is one that preserves flexibility along both these dimensions at all first stage states reached at time $t+1$. An irreversible decision leads to sure loss of options with respect to initial availability of options and perhaps a loss of options at a state which is also reached by a reversible decision. But the definition of an irreversible decision does not preclude the creation of new options *vis à vis* the original availability at any of the states or *vis à vis* the reversible decision at some state that cannot be reached by the reversible decision. For these reasons, an irreversible decision is not just the opposite of a reversible decision but something stronger and therefore needed to be defined separately. There can be decisions which are neither reversible nor irreversible but they are not considered here because of our interest in showing the connection between the various existing models and the irreversibility effect; no doubt they can be accommodated by change of terminology (total vs partial reversibility for instance).

3.4. *Exogeneous vs Endogeneous Uncertainty*

Uncertainty is said to be exogeneous when:

$$P^o_{a_i}(d_t) = P^o_{a_i}(\bar{d}_t) \quad \text{for all } a_i, i = 1, 2, \ldots, I \text{ in } \mathscr{A} \text{ and all decisions } d_t, \bar{d}_t \in D_t(o)\,.$$

If uncertainty is not exogeneous then it is endogeneous.

Thus exogeneous uncertainty is as if uncertainty is given by nature for all the decisions available at the initial time period t and is the same for all of them.

Now in order to show the connection between the various models of the irreversibility effect we make explicit the following assumption.

ASSUMPTION. It is assumed that there is only one source of uncertainty and that $D_{t+1}(a_i \mid r_t) = D_t(o)$ for all first stage states a_i, $i = 1, 2, \ldots, I$.

3.5. *Comment*

Under the above assumption when uncertainty is exogeneous:

$$D_{t+1}(a_i \mid ir_t) \subseteq D_{t+1}(a_i \mid r_t) = D_t(o) \quad \text{for all states } a_i,\ i = 1, 2, \ldots, I\,;$$

$$D_{t+1}(a_{i^*} \mid ir_t) \subset D_{t+1}(a_{i^*} \mid r_t) = D_t(o) \quad \text{for at least one state } a_{i^*}$$

i.e. an irreversible decision leads to sure loss of flexibility over time as well as with respect to the reversible decision. Note that when uncertainty is exogeneous every state that can be reached by the irreversible decision can also be reached by the reversible decision. Then from the definitions of reversible and irreversible decisions the above comment follows.

4. MODELS OF THE IRREVERSIBILITY EFFECT

As we said before the problem of the decision maker is to choose at a time t, and at an initial state o, a decision for time t and a vector of decisions for time $t+1$ for each of the information sets he envisages at time $t+1$. Now, between time t and $t+1$ there might be occasion to obtain more information. In other words between the time that he executes the first decision and the time he executes the last decision, the decision maker might obtain more information. In this case his choice at time t would depend on not only his present calculations about the utility payoffs associated with each of the decisions but also the anticipated gains associated with more information. In this context the irreversibility effect was presented to weigh the present advantages of irreversible decisions against anticipated gains from information.

4.1. The Irreversibility Effect

Let information structure $S_{\hat{\theta}}$ be a refinement of information structure S_θ then:
if

$$\pi(r_t, S_\theta) \geqslant \pi(ir_t, S_\theta)$$

it implies

$$\pi(r_t, S_{\hat{\theta}}) \geqslant \pi(ir_t, S_{\hat{\theta}})\,;$$

whereas if

$$\pi(ir_t, S_\theta) \geqslant \pi(r_t, S_\theta)$$

it does not imply

$$\pi(ir_t, S_{\hat{\theta}}) \geqslant \pi(r_t, S_{\hat{\theta}}) .$$

Thus the irreversibility effect states that if under a particular information structure a reversible decision dominates an irreversible decision, in the sense of yielding greater payoffs, then the reversible decision will continue to yield greater payoffs under a refinement of the original information structure. However, if an irreversible decision dominates a reversible decision under a particular information structure then it may not continue to do so with more information.

Henry (1974) and Laffont and Freixas (1984) (and all the applied papers) considered uncertainty to be exogeneous. The comment given earlier then explains why they considered irreversibility to be synonymous with loss of flexibility. In their models under exogeneous uncertainty, the irreversible decisions could not reach any state and could not make available any new option other than the ones made available by the reversible decision. Moreover, Henry who stated the original irreversibility effect and proved it by an example considered the case where an irreversible decision led to a total loss of flexibility. For instance, d_t could be either 0 or 1 and d_{t+1} had to be either 0 or 1 and greater than or equal to d_t. Thus, if the number 1 was chosen at time t, it had to be chosen at time $t+1$ also. He also considered only two possible information structures, incomplete information and complete information. Laffont and Freixas (1984) generalized the information structure to include all possibilities between complete and incomplete information. Then they considered a particular case of continuous decisions (d_t and d_{t+1} were intervals rather than numbers). Here they showed that a necessary condition for the irreversibility effect was quasi-concavity of the expected utility function.

5. THEORETICAL RESULTS

We now present a restatement of the irreversibility effect as Proposition 1 where we consider uncertainty to be exogeneous but without assuming any particular form of dependence between d_t and d_{t+1}. Here an irreversible decision shrinks the space of options, but not necessarily to a single point.

5.1. Decision Dominance

A decision d_t is said to dominate {*strictly dominate*} a decision $\bar{d}_t$ under information structure S_θ if $\pi(d_t, S_\theta) \geqslant \{>\} \pi(\bar{d}_t, S_\theta)$.

5.2. PROPOSITION 1. *Suppose* $P^o_{a_i}(r_t) = P^o_{a_i}(ir_t)$ *for all* a_i, $i = 1, 2, \ldots, I$ *in* $\mathscr{A}$ *then*:

(i) *Without further restrictions the irreversibility effect need not hold.*
(ii) *If* $U(r_t) > U(ir_t)$ *then the reversible decision will strictly dominate under all information structures.*
(iii) *If* $U(r_t) \leqslant U(ir_t)$ *and the reversible* {irreversible} *decision dominates under an information structure* S_θ *then the reversible* {irreversible} *decision will continue to dominate under* $S_{\hat{\theta}}$ *a refinement of* S_θ *if and only if,*

$$\alpha(r_t, S_\theta, S_{\hat{\theta}}) \geqslant \alpha(ir_t, S_\theta, S_{\hat{\theta}}) \cdot$$
$$\{\alpha(\mathrm{ir}_t, \mathrm{S}_\theta, \mathrm{S}_{\hat{\theta}}) \geqslant \alpha(\mathrm{r}_t, \mathrm{S}_\theta, \mathrm{S}_{\hat{\theta}})\}$$

where $\alpha(d_t, S_\theta, S_{\hat{\theta}})$ *is the increase in payoffs for decision* d_t *from gain in information in going from* S_θ *to* $S_{\hat{\theta}}$ *or* $\{V(d_t, d^*_{t+1}(d_t), S_{\hat{\theta}}) - V(d_t, d^*_{t+1}(d_t), S_\theta)\}$.

Proof. (i) As proof we give the following counter example. Consider a decision tree with initial node I_0, two first stage nodes a_1, a_2 and two second stage nodes b_1, b_2, where the structure of states, the availability of decisions, the probability matrices and the structure of rewards are as follows.

Availability of decisions:

$$D_t(I_0) = \{d_t, \bar{d}_t\},$$
$$D_{t+1}(a_1 \mid d_t) = D_{t+1}(a_2 \mid d_t) = \{d_{t+1}, \bar{d}_{t+1}\},$$
$$D_{t+1}(a_1 \mid \bar{d}_t) = \{d_{t+1}\}, \quad D_{t+1}(a_2 \mid \bar{d}_t) = \{d_{t+1}, \bar{d}_{t+1}\}.$$

Probability matrix for decision d_t:

$$P_{a_1}^{I_0}(d_t) = 0.1\,; \qquad P_{a_2}^{I_0}(d_t) = 0.9\,;$$
$$P_{b_1}^{a_1}(d_{t+1}) = 0.5\,; \quad P_{b_2}^{a_1}(d_{t+1}) = 0.5\,;$$
$$P_{b_1}^{a_2}(d_{t+1}) = 0.6\,; \quad P_{b_2}^{a_2}(d_{t+1}) = 0.4\,.$$

Probability matrix for decision $\bar{d}_t$:

$$P_{a_1}^{I_0}(\bar{d}_t) = 0.1\,; \qquad P_{a_2}^{I_0}(\bar{d}_t) = 0.9\,;$$
$$P_{b_1}^{a_1}(\bar{d}_{t+1}) = 0.9\,; \quad P_{b_2}^{a_1}(\bar{d}_{t+1}) = 0.1\,;$$
$$P_{b_1}^{a_2}(\bar{d}_{t+1}) = 0.4\,; \quad P_{b_2}^{a_2}(\bar{d}_{t+1}) = 0.6\,.$$

Note that the figures given for $P_{b_j}^{a_i}(d_{t+1})$, $P_{b_j}^{a_i}(\bar{d}_{t+1})$ hold when d_{t+1} and $\bar{d}_{t+1}$, respectively, are available at a_i as a consequence of the decision taken at time t. When they are not available the corresponding probabilities are zero.

Intermediate rewards associated with decisions:

$$U(d_t) = 1\,, \quad U(\bar{d}_t) = 2\,.$$

Final rewards associated with terminal states:

$$\mathcal{U}'(b_1) = 10\,, \quad \mathcal{U}'(b_2) = 20\,.$$

Evidently decision d_t is the reversible decision and decision $\bar{d}_t$ is the irreversible decision. Since there are only two first stage states, there can be only two types of information structures, incomplete information indicated by S_θ and complete information indicated by $S_{\hat{\theta}}$. Let $IC_{d,d}$ represent expected terminal payoffs under incomplete information for the sequence of decisions d_t, d_{t+1} taken at times t and $t+1$, respectively, and similarly for other sequences of decisions. Then for the above example we have the following:

$$
\begin{aligned}
IC_{d,d} &= 0.1 \times (0.5 \times 10 + 0.5 \times 20) \\
&\quad + 0.9 \times (0.6 \times 10 + 0.4 \times 20) = 14.1 ; \\
IC_{d,\bar{d}} &= 0.1 \times (0.9 \times 10 + 0.1 \times 20) \\
&\quad + 0.9 \times (0.4 \times 10 + 0.6 \times 20) = 15.5 ; \\
IC_{\bar{d},d} &= 0.1 \times (0.5 \times 10 + 0.5 \times 20) \\
&\quad + 0.9 \times (0.6 \times 10 + 0.4 \times 20) = 14.1 ; \\
IC_{\bar{d},\bar{d}} &= 0.9 \times (0.4 \times 10 + 0.6 \times 20) = 14.4 .
\end{aligned}
$$

As before, let $d^*_{t+1}(d_t)$ be the best response in time period $t+1$ to the decision d_t taken at time period t. Thus under incomplete information:

$$
\begin{aligned}
&V(d_t, d^*_{t+1}(d_t), S_\theta) = \mathrm{Max}(IC_{d,d}, IC_{d,\bar{d}}) = 15.5 ; \\
&V(\bar{d}_t, d^*_{t+1}(\bar{d}_t), S_\theta) = \mathrm{Max}(IC_{\bar{d},d}, IC_{\bar{d},\bar{d}}) = 14.4 ; \\
&\pi(d_t, S_\theta) = 16.5 > 16.4 = \pi(\bar{d}_t, S_\theta) .
\end{aligned}
$$

Under complete information, $d_{t+1}(d_t)$ also depends on the state a_i that has been reached. Thus:

$$
\begin{aligned}
&V(d_t, d^*_{t+1}(d_t), S_{\hat{\theta}}) = 0.1 \times (15) + 0.9 \times (16) = 15.9 ; \\
&V(\bar{d}_t, d^*_{t+1}(\bar{d}_t), S_{\hat{\theta}}) = 0.1 \times (15) + 0.9 \times (16) = 15.9 ; \\
&\pi(d_t, S_{\hat{\theta}}) = 16.9 < 17.9 = \pi(\bar{d}_t, S_{\hat{\theta}}) .
\end{aligned}
$$

Hence the irreversibility effect is refuted under these circumstances.

(ii) Recall that the rewards of a decision d_t is the sum of an intermediate reward and a final reward. The final reward from any decision in a two stage decision making process is a weighted average of the payoffs at each first stage information set (which in turn are a function of the second decision used at each information set), the weights being given by the probability density of the first decision to the first stage states. Thus two factors determine the final payoff, the actual rewards at the first stage information sets and the weights.

Under uncertainty given by nature the weights are the same for both the decisions.

Thus when $P^o_{a_i}(r_t) = P^o_{a_i}(ir_t)$, it follows from the comment that for all information structures S_θ we have:

$$V(r_t, d^*_{t+1}(r_t), S_\theta) \geqslant V(ir_t, d^*_{t+1}(ir_t), S_\theta) .$$

This is true by definition of a reversible and an irreversible decision and by definition of $V(d_t, d^*_{t+1}(d_t), S_\theta)$. From the above and the fact that $\pi(d_t, S_\theta) = U(d_t) + V(d_t, d^*_{t+1}(d_t), S_\theta)$ the proposition is obvious.

(iii) Note that for any information structure S_θ and its refinement $S_{\hat{\theta}}$ the payoffs may be written as:

$$\pi(r_t, S_{\hat{\theta}}) = \pi(r_t, S_\theta) + \alpha(r_t, S_\theta, S_{\hat{\theta}}) .$$

From the above fact the proposition follows. Though this may seem obvious to the reader, the above statement was made in order to highlight the fact that the gains from more information need not be an increasing function of the availability of options; i.e. it need not be more for the reversible function. For part (i) of this proof we had given an example where the gains from more information had been less for a reversible decision *vis à vis* an irreversible decision; but by changing the probability weights of that same example we can arrive at a situation where the inequality sign is reversed.

Hence it can be noted that the irreversibility effect can occur only in the case where $U(r_t) \leqslant U(ir_t)$ and $\alpha(r_t, S_\theta, S_{\hat{\theta}}) > \alpha(ir_t, S_\theta, S_{\hat{\theta}})$. Now we can also see how the example of Henry worked. In his example an irreversible decision reduces the space of future options to a single point. So there is no gain from more information for the irreversible decision whereas there could be a gain from more information for the reversible decision. Applying Proposition 1 to this particular case gives the irreversibility effect.

5.3. PROPOSITION 2. *When uncertainty is intrinsic or $P^o_{a_i}(r_t) \neq P^o_{a_i}(ir_t)$ the irreversibility effect need not hold; moreover we cannot predict, under which situation a decision will be dominant using the notion of irreversibility.*

Proof. We illustrate the first part by means of the following example. We consider the same decision tree and the availability of decisions as before. But we change the intermediate rewards as follows; $U(d_t) = U(\bar{d}_t) = 1$. The probability matrices and the rewards are given below:

Probability matrix for decision d_t:

$$P_{a_1}^{I_0}(d_t) = 0.8\,; \quad P_{a_2}^{I_0}(d_t) = 0.2\,;$$
$$P_{b_1}^{a_1}(d_{t+1}) = 0.5\,; \quad P_{b_2}^{a_1}(d_{t+1}) = 0.5\,;$$
$$P_{b_1}^{a_2}(d_{t+1}) = 0.6\,; \quad P_{b_2}^{a_2}(d_{t+1}) = 0.4\,.$$

Probability matrix for decision $\bar{d}_t$:

$$P_{a_1}^{I_0}(\bar{d}_t) = 0.1\,; \quad P_{a_2}^{I_0}(\bar{d}_t) = 0.9\,;$$
$$P_{b_1}^{a_1}(\bar{d}_{t+1}) = 0.9\,; \quad P_{b_2}^{a_1}(\bar{d}_{t+1}) = 0.1\,;$$
$$P_{b_1}^{a_2}(\bar{d}_{t+1}) = 0.4\,; \quad P_{b_2}^{a_2}(\bar{d}_{t+1}) = 0.6\,.$$

Thus we have:

$$\pi(d_t, S_\theta) = 15.8 > 15.4 = \pi(\bar{d}_t, S_\theta)\,;$$
$$\pi(d_t, S_{\hat{\theta}}) = 16.2 < 16.9 = \pi(\bar{d}_t, S_{\hat{\theta}})\,;$$
$$\alpha(d_t, S_\theta, S_{\hat{\theta}}) = 0.4 < 1.5 = \alpha(\bar{d}_t, S_\theta, S_{\hat{\theta}})\,.$$

Let us start with incomplete information when there is only one information set. When a decision is reversible it has the advantage that at every first stage node which can be reached by both the reversible and irreversible decision, the decisions available as a consequence of using the irreversible decision are also available as a consequence of using the reversible decision. This means that, at a node that can be reached by both kinds of decisions, the rewards that are made possible by using an irreversible decision can also be enjoyed by employing the reversible decision. However, the expected final return $V(\cdot)$ from a decision is a weighted average of rewards at the first stage nodes, with the weights being the probabilities to the respective nodes. Thus it is possible, that from the entire information set, the payoff to the irreversible decision is greater because the probability weights favour the irreversible decision (i.e. $P^o_{a_i}(ir_t)$ might stochastically dominate $P^o_{a_i}(r_t)$).

Moreover, there might be states or information sets that may be created by the use of the irreversibile decision, which can only be reached by using the irreversible decision, and the benefits of which are available only as a consequence of using the irreversible decision. Therefore when uncertainty is not the same for all decisions, under incomplete or partial information there is no possibility of using the notion of irreversibility of a decision to outline general conditions under which $V(r_t, d^*_{t+1}(r_t), S_\theta)$ is bigger than $V(ir_t, d^*_{t+1}(ir_t), S_\theta)$ or $\pi(r_t, S_\theta)$ is bigger than $\pi(ir_t, S_\theta)$.

The total payoffs for any other information structure $S_{\hat{\theta}}$ may be written as:

$$\pi(r_t, S_{\hat{\theta}}) = \pi(r_t, S_\theta) + \alpha(r_t, S_\theta, S_{\hat{\theta}}) .$$

The definition holds similarly for the irreversible decision. As we have seen before it is not necessary for the gains from information to be more for a reversible or for an irreversible decision. Everything depends on the structure of rewards and the structure of decisions for that particular case. Thus we can not arrive at rules or propositions to predict which decision will be dominant. All one can say here is that the impact of a decision will be a function of the probability matrix associated with it, the reward structure, the flexibility of the decision and the gains from information.

6. CONCLUSION

In this paper we examined the logical construct necessary to justify a normative proposition termed the irreversibility effect. The irreversibility effect pertained to the relation between irreversibility, uncertainty and gains from information in a two period (t and $t+1$) decision making model. The proposition cautioned decision makers against taking irreversible decisions if they anticipated getting more information between the time of taking the first decision and time of taking the second and final decision.

In this context we first examined the relation between flexibility and irreversibility. It was shown that an irreversible decision led to loss of flexibility *vis à vis* the initial availability of decisions and *vis à vis* a reversible decision when uncertainty was exogeneous or the same for all decisions at time period t. Otherwise an irreversible decision led to loss of flexibility *vis à vis* the initial availability of decisions but retained the possibility of making new options available.

Then, in Propositions 1 and 2, it was shown that the irreversibility effect could hold only when an irreversible decision led to total loss of flexibility in all future states. Outside of this special case, when uncertainty was exogeneous some predictions could be made on the optimal decision at time t, if one had certain information like the intermediate rewards associated with the decisions. However, when uncertainty was endogeneous no predictions could be made using the notion of irreversibility.

Essentially the irreversibility effect failed in the general case because of the counter intuitive fact that the gains from anticipated information need not always be more for the reversible as compared to the irreversible decision. There is no obvious connection between flexibility and the gains from anticipated information.

Thus our conclusion is a normative statement about a normative statement, namely: do not reject a decision on the basis of its irreversibility; make a thorough analysis of all the externality effects; take into account all the new options that an irreversible decision can create (for it can create options not possible with a reversible decision). This argument is valid for a number of current debates in economic policy. For instance whether or not to allow commercialization of transgenic plants. Most antagonists worry about the irreversible nature of this decision; whereas many proponents justify its rationality on the basis of the new options that this decision could open up.

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REFERENCES

Arrow, K.J. and Fisher, A.C.: 1974, 'Environmental preservation, uncertainty and irreversibility', *Quarterly Journal of Economics*, **88**, 312–320.

Bernanke, B.J.: 1983, 'Irreversibility, uncertainty and cyclical investment', *Quarterly Journal of Economics*, **98**, 85–106.

Cohendet, P. and Llerena, P. (eds.): 1989, *Flexibilite, information et decision*, Economica, Paris.

Fisher, A.C. and Hanemann, W.M.: 1990, 'Option Value: theory and measurement', *European Review of Agricultural Economics*, **17**, 167–180.

Fisher A.C. and Krutilla, J.: 1974, 'Valuing long run ecological consequences and irreversibilities', *Journal of Environmental Economics and Management*, **1**, 96–108.

Freixas, X.: 1986, L'effet-irreversibilite generalise, in: Boiteux M., De Montbrial, T., and Munier, B. (Eds.), *Marches, Capital et Incertitude, Essais en l'honneur de Maurice Allais*, Economica, Paris.

Freixas, X.: 1987, 'L'effet-irreversibilité dans le choix de grands projects', *Revue Economique*, **38**, 149–155.

Freixas, X. and Laffont, J.J.: 1984, 'On the irreversibility effect', in: Kihlstrom R.E. and Boyer, M. (Eds.), *Bayesian Models in Economic Theory*, North Holland, Amsterdam, pp. 105–114.

Hanemann, W.M.: 1989, 'Information and the concept of option value', *Journal of Environmental Economics and Management*, **16**, 23–37.

Henry, C.: 1974a, 'Investment decisions under uncertainty: The 'irreversibility effect,' *American Economic Review*, **64**, 1006–1012.

Henry, C.: 1974b, 'Option values in the economics of irreplaceable assets', *Review of Economic Studies*, 89–104.

Jones, R.A. and Ostroy, J.M.: 1984, 'Flexibility and uncertainty', *Review of Economic Studies*, **51**, 13–32.

Kreps, D.: 1979, 'A representation theorem for "preference for flexibility",' *Econometrica*, **47**, 565–577.

Pindyck, R.S., 1991, 'Irreversibility, uncertainty and investment,' *Journal of Economic literature*, **29**(3), 1110–1148.

Richard, A.: 1982, 'Elements de synthese entre valeur actualisee et delai de recuperation, "l'effet irreversibilite",' *Revue d'Economie Politique*, **92**(1), 1–15.

Richard, A.: 1984, 'La demande de monnaie: Une synthese fondee sur l'irreversibilite et le risque', *Revue d'Economie Politique*, **94**, 202–225.

Usategui, J.M.: 1990, 'Uncertain irreversibility, information and transformation costs', *Journal of Environmental Economics and Management*, **19**, 73–85.

Viscusi, W.K.: 1985, 'Environmental policy choice with an uncertain chance of irreversibility', *Journal of Environmental Economics and Management*, **12**, 28–44.

Viscusi, W.K.: 1988, 'Irreversible environmental investments with uncertain benefit levels', *Journal of Environmental Economics and Management*, **15**, 147–157.

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