Risk Aversion and the Efficiency Wage Contract

Ouassila Chouikhi — Shyama V. Ramani

Abstract. The efficiency wage model of Shapiro and Stiglitz (*American Economic Review* 74: 433–444, 1984) has not always been confirmed by empirical investigations. This could be due to informational problems. Reformulating the Shapiro and Stiglitz model as a sequential game, this paper examines the relations between the terms of the efficiency wage contract offered by a firm and the responses of a worker, under incomplete information about the degree of risk aversion of the firm and the worker. It shows that under incomplete information about the degree of risk aversion of the worker, shirking can emerge as an equilibrium phenomenon. For any efficiency wage contract, a worker will shirk if the degree of risk aversion of the worker is less than that corresponding to the contract.

1. Introduction

One of the explanations proposed by economists for the existence of a non-clearing labour market is that the wages offered by the firms are above the market-clearing Walrasian wage. Starting with the seminal article by Shapiro and Stiglitz (1984), a number of models have shown that such a high wage could be an 'efficiency wage' necessary to eliminate shirking or opportunistic behaviour on the part of workers, thus providing a strategic microeconomic

---

Ouassila Chouikhi, IREPD, University Pierre Mendès France, BP 47, 38040 Grenoble cedex 9, France. E-mail: ouassila.chouikhi@upmf-grenoble.fr.

Shyama V. Ramani (author for correspondence), INRA, University Pierre Mendès France, BP 47, 38040 Grenoble cedex 9, France. Tel: +33 (0)4 76 82 54 12; Fax: +33 (0)4 76 82 54 55; E-mail: shyamar@grenoble.inra.fr.

We would like to thank Lamia Mansouri, Serge Leo and the participants at the Economic Science Association, European Regional Meeting, for their useful comments. We are also extremely grateful for the insightful comments made by an anonymous referee. We remain responsible for all errors.

LABOUR 18 (1) 53–73 (2004) JEL C72, J41

justification for this phenomenon (for surveys see Akerlof and Yellen, 1986; Katz, 1986; Weiss, 1991; Yellen, 1984; Zenou, 1996). These models have proved that when a labour contract is imperfectly enforceable due to the non-observability of effort and imperfect monitoring, it is more profitable for a firm to offer a higher salary and eliminate shirking, even in the presence of involuntary unemployment. However, this theory has not always been confirmed by empirical investigations. As a possible explanation we will show in this paper that shirking can emerge as an equilibrium phenomenon under incomplete information about the degree of risk aversion of the worker.

The efficiency wage models have been set in two main contexts. Under the first, there is only a problem of moral hazard, i.e. the firm has imperfect information on the effort made by the worker, but all the characteristics of the workers are common knowledge. Here, the efficiency wage serves to eliminate opportunism by offering workers a premium for respecting the contract (e.g. Bowles, 1985; Shapiro and Stiglitz, 1984; Sparks, 1986). Under the second, there is not only a problem of moral hazard but also one of adverse selection. This implies that the monitoring of effort is imperfect and, in addition, the firm has incomplete information on the characteristics of the worker (e.g. innate competence in addition to certified qualifications). In this case, *if* the market can screen the workers and make them reveal their true nature, then the appropriate efficiency wage contract, corresponding to the workers' true nature or type, can be offered and shirking can be eliminated at equilibrium (Strand, 1987).[1] On the other hand, if screening is not possible, then the firm has to offer a contract based upon its beliefs about the true nature of the worker. In such cases shirking may sometimes occur when an inappropriate contract is offered to a worker. In the literature on efficiency wages there has been very little study of the different sources of incomplete information and the corresponding conditions which can lead to shirking as an equilibrium phenomenon. Herein lies our contribution.

The efficiency wage contract is defined as the profit-maximizing effort–wage combination that eliminates shirking by workers. However, under incomplete information, it may not be effective as a screening device. In this case, any other screening mechanism that also eliminates shirking will be more costly, as the firm will have to pay a premium in order to induce the workers to reveal their private information. Therefore, it may not always be optimal for a firm to implement a screening mechanism that separates different types of workers. Under certain circumstances a firm may prefer to offer a set of efficiency wage contracts which does not screen workers, eliminating opportunism by certain types of workers but not by others (Mas-Collel *et al.*, 1995). The present paper is assumed to be situated in such a context.[2] Then, reformulating the Shapiro and Stiglitz model as a sequential game, with incomplete information on the players' degree of risk aversion, it attempts to answer three central questions:

- How do the terms of the efficiency wage contract vary with the degrees of risk aversion of the firm and the worker?
- What is the equilibrium contract under complete and incomplete information?
- Under which conditions can shirking be observed at equilibrium?

The contribution of this paper to the theoretical literature on the efficiency wage contract is threefold. Firstly, it *gives some additional insight on the working of the shirking model*. When the Shapiro and Stiglitz model is extended by incorporating the players' degrees of risk aversion, under complete information, the terms of the efficiency wage contract are independent of the firm's degree of risk aversion, but they vary as a function of the worker's degree of risk aversion. Furthermore, for any effort, the corresponding efficiency wage decreases with the worker's degree of risk aversion. However, under incomplete information the type of contract offered by the firm is influenced by the degree of risk aversion of the firm.

Secondly, under incomplete information, the results identify the conditions under which *shirking occurs at equilibrium, even after the efficiency wage contract has been accepted.* According to the model, shirking occurs at equilibrium, whenever the firm offers an efficiency wage contract such that the degree of risk aversion of the worker is less than that corresponding to the contract.

Finally, it shows that *the source of incomplete information considered matters*. Our results differ considerably from those of Albrecht and Vroman (1998), who consider incomplete information with respect to the worker's disutility of effort (as will be detailed in the next section).

The remainder of this article is organized as follows. Section 2 discusses the literature on shirking as an equilibrium phenomenon. Section 3 presents our extension of the Shapiro and Stiglitz model. Section 4 contains the results on the efficiency wage contract under complete and incomplete information. Finally, Section 5 presents the conclusions.

2. Shirking as a possible equilibrium phenomenon

The original Shapiro and Stiglitz model has been tested empirically with panel data on firms and through laboratory experiments. Cappelli and Chauvin (1991) checked the shirking model of efficiency wages by examining the relationship between the rates of employee discipline and relative wage premiums across plants within the same firm. Their results suggest that greater wage premiums are associated with lower levels of shirking, as measured by disciplinary dismissals. One drawback of testing the shirking model using panel data is that the information used concerns the response of workers to contracts that have been accepted, and they do not reveal how many or what types of contracts have been rejected by workers. This is indeed the advantage of the experimental method, which permits the testing of theoretical predications by replicating the theoretical context and generating data on the entire sequence of decisions forming the theory.

Fehr *et al*. (1996) tested the Shapiro and Stiglitz model by conducting a series of efficiency wage experiments. Assuming identical costs of effort for all workers, they found that higher wages caused a reduction in shirking. However, among the contracts proposed by firms respecting the no-shirking condition, about 13 per cent were not respected by the workers. In other words, shirking was observed even after the worker had accepted the efficiency wage contract.

Albrecht and Vroman (1998) provide a theoretical explanation for shirking after an efficiency wage contract is accepted by considering contract initiation under incomplete information, where a worker's disutility of effort is private information unknown to the firm. They show that whenever the disutility of effort stipulated in the contract is lower than the real disutility of effort of the worker, shirking emerges as an equilibrium phenomenon.

The present paper incorporates another kind of heterogeneity, in terms of the degree of risk aversion of the players, and identifies another context in which shirking can occur at equilibrium. Furthermore, although the two models are not comparable, as the strategy spaces and the payoff functions of the players are different in the two models, it is worth noting that the central result on the equilibrium strategies of firms is also different. In the Albrecht and

Vroman model, identical firms never offer the same efficiency wage contract at equilibrium (i.e. firms play asymmetric pure strategies). Whereas in our model, if firms are identical, i.e. if they all have common beliefs and the same degrees of risk aversion, then they play the same pure strategy, or offer the same contract, at equilibrium.

3. Model

Consider a labour market where contract initiation between a firm and a worker takes place in a sequential game. Firms and workers can be one of three possible types, depending upon their degree of risk aversion, i.e. they can be either risk averse, risk neutral or risk loving. The economy contains a positive proportion of each type of agent and these ratios are common knowledge to all players.

The objective of the firm and the worker is to maximize their utility. The utility functions of the firm and the worker are given by U_f and U_l, respectively, and they are strictly increasing in the payoffs of the firm and the worker, Π_f and Π_l, respectively. Furthermore, the payoffs of the firm and the worker, Π_f and Π_l, are functions of the contract offered by the firm C_f and the effort rendered by the worker e (i.e. Π_f (C_f, e); Π_l (C_f, e)). The degree of risk aversion of the firm is given by δ_i, and the degree of risk aversion of the worker is given by β_i, where i can be either RA standing for a risk-averse, RN for a risk-neutral and finally RL for a risk-loving agent. For a risk-averse firm and worker $1 > \delta_{RA} > 0$ and $1 > \beta_{RA} > 0$, for a risk-neutral firm or worker $\delta_{RN} = \beta_{RN} = 0$, and finally for a risk-loving firm or worker $\delta_{RL} < 0$ and $\beta_{RL} < 0$.[3] Thus, we have:

$$U_f(e,w) = \Pi_f(e,w)^{1-\delta}$$

and

$$U_l(e,w) = \Pi_l(e,w)^{1-\beta}.$$

The above arguments also confirm that when agents are risk averse their utility functions are strictly concave in payoffs, when they are risk loving their utility functions are strictly convex in payoffs, and when they are risk neutral their utility functions are linear in payoffs.

The game of contract initiation between a firm and a worker begins with nature choosing the type of firm and the type of worker. Then the firm moves by proposing a contract, C_f, to the worker, where C_f consists of an effort e and a wage associated with the application of this effort, w. The effort e is chosen from the closed interval $[e_0, e^{\max}]$.

The worker plays next and can either accept or refuse the contract C_f. If he rejects it, he remains unemployed and gets an unemployment benefit of b. However, if the worker accepts the contract, then he can decide whether or not to respect it. Respecting the contract implies that he will supply the effort as dictated by the contract; not respecting it means that he will shirk and apply a lower effort. For any effort e supplied, the worker bears an effort cost, $c(e)$, where $c(e)$ is an increasing and convex function of e.

In order to discourage shirking, the firm has a monitoring system that can detect shirking by a worker with an exogenous probability s. If a worker is found to be shirking then he is paid a positive sum g, which is always less than the wage w. Since the reduction in payoffs to the worker (i.e. $w - g$) is independent of the level of shirking, if the worker chooses to shirk, then he applies the least effort e_0. Thus, whenever a worker shirks, his payoff is a lottery, where he either gets $g - c(e_0)$ with a probability s or $w - c(e_0)$ with a probability $(1 - s)$. Then the expected payoff and the utility of the worker, when he accepts and respects a contract, or when he shirks, can be indicated as shown by equations [1] and [2], respectively:

$$\Pi_l(e,w) = w - c(e) \Leftrightarrow U_l(e,w) = (w - c(e))^{1-\beta} \qquad [1]$$

$$\begin{aligned}\Pi_l(e,w) &= s(g - c(e_0)) + (1-s)(w - c(e_0)) \Leftrightarrow U_l(e,w) \\ &= s(g - c(e_0))^{1-\beta} + (1-s)(w - c(e_0))^{1-\beta}. \end{aligned} \qquad [2]$$

When a firm offers a contract $C_f = (e, w)$ to a worker, if the worker accepts the contract and respects it in terms of the effort supplied, then the firm earns a revenue qe (where $q > 0$) and bears the wage cost w. The parameter q is equal to the constant marginal productivity of the worker's efforts and can be interpreted as the firm's production technology. If the worker shirks, then the firm faces a lottery. The revenue generated is qe_0, but the wage paid by the firm depends on whether or not the worker is caught shirking. If the worker is detected shirking, then the firm pays the worker g, otherwise it pays the worker w. The profit of the firm, Π_f, and its utility from such payoffs, U_f, when the worker respects or shirks on the contract are given by equations [3] and [4], respectively:

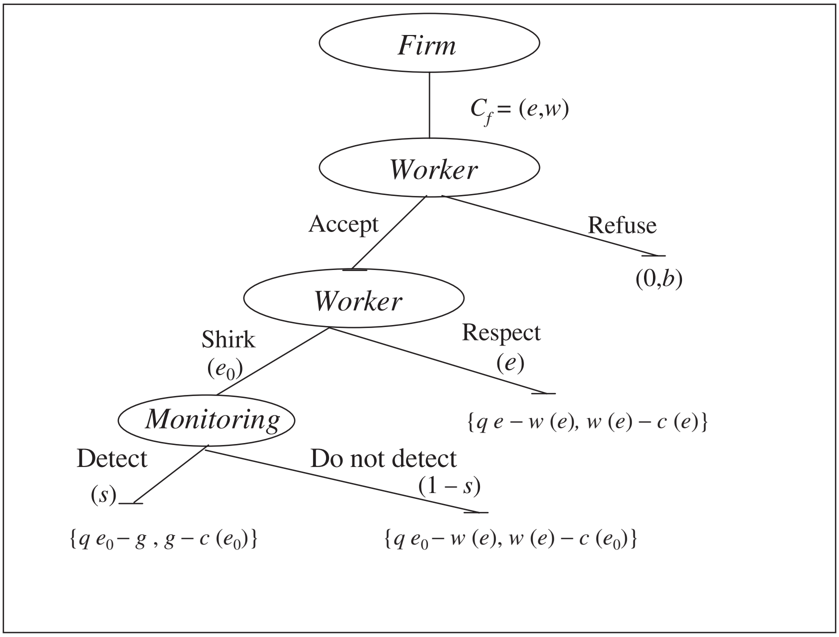
$$\Pi_f(e,w) = qe - w \Leftrightarrow U_f(e,w) = (qe - w)^{1-\delta} \quad [3]$$

$$\Pi_f(e,w) = s(qe_0 - g) + (1-s)(qe_0 - w) \Leftrightarrow U_f(e,w) = s(qe_0 - g)^{1-\delta} + (1-s)(qe_0 - w)^{1-\delta}. \quad [4]$$

This finishes our presentation of the game, which is also illustrated in Figure 1 for the complete information case.

Now, let us turn to the solution of the game proceeding by the usual method of backward induction. A worker will accept a contract $C_f = (e, w)$ if the utility he gets from it is at least as much as from the unemployment benefit, b. This is also referred to as the participation constraint for contract initiation and is given by the following equation:

Figure 1. Labour contract game under complete information



$$w = c(e) + b \Leftrightarrow w^{1-\beta} = (c(e) + b)^{1-\beta}.$$

There is a problem of incentives if for any contract satisfying the participation constraint: (i) a worker prefers to shirk rather than respect the contract; and (ii) this shirking leads to a loss for the firm. As the following lemma will indicate, this is indeed the case by assuming that $c(e^{\max}) \leq (g - b)$.

Lemma 1: The incentive problem is binding if $c(e^{\max}) \leq (g - b)$.

From equations [3] and [4], we can infer that for any contract satisfying the participation constraint, the utility of the firm will be lower when the worker shirks, if:

$$(qe - w)^{1-\delta} > s(qe_0 - g)^{1-\delta} + (1 - s)(qe_0 - w)^{1-\delta}.$$

Substituting $w = c(e) + b$, we get:

$$\Rightarrow (qe - c(e) - b)^{1-\delta} > s(qe_0 - g)^{1-\delta} + (1 - s)(qe_0 - c(e) - b)^{1-\delta}. \qquad [5]$$

Note that $(qe - c(e) - b)^{1-\delta} \geq (qe_0 - c(e) - b)^{1-\delta}$. Therefore, if we have following:

$$(qe_0 - c(e) - b)^{1-\delta} > s(qe_0 - g)^{1-\delta} + (1 - s)(qe_0 - c(e) - b)^{1-\delta}, \qquad [6]$$

then equation [5] is satisfied. From equation [6], we have:

$$s(qe_0 - c(e) - b)^{1-\delta} > s(qe_0 - g)^{1-\delta} \Leftrightarrow qe_0 - c(e) - b$$
$$> qe_0 - g \Leftrightarrow c(e) < g - b.$$

From the above, we can deduce that if $c(e) < g - b$ then the firm's profit will be lower whenever the worker shirks.

Let us now turn to the worker and examine the condition under which he will shirk. From equations [1] and [2] we know that for any contract satisfying the participation constraint, a worker will obtain a higher utility from shirking if:

$$(w - c(e))^{1-\beta} < s(g - c(e_0))^{1-\beta} + (1 - s)(w - c(e_0))^{1-\beta}.$$

Substituting $w = c(e) + b$ in the above, we have:

$$b^{1-\beta} < s(g - c(e_0))^{1-\beta} + (1 - s)(c(e) + b - c(e_0))^{1-\beta}. \qquad [7]$$

Note that $b \leq c(e) + b - c(e_0)$. This means that if we suppose the following:

$$b^{1-\beta} < s(g - c(e_0))^{1-\beta} + (1-s)b^{1-\beta}, \qquad [8]$$

then we get:

$$b^{1-\beta} < s(g - c(e_0))^{1-\beta} + (1-s)b^{1-\beta} \leq s(g - c(e_0))^{1-\beta} + (1-s)(c(e) + b - c(e_0))^{1-\beta}.$$

Thus, equation [7] will always be satisfied if equation [8] holds. From [8] we get:

$$s(b)^{1-\beta} < s(g - c(e_0))^{1-\beta} \Leftrightarrow c(e_0) < g - b.$$

Therefore, equation [7] is always satisfied if $c(e_0) < g - b$, in which case the worker will prefer to shirk.

Thus, there is a problem of incentives if $c(e_0) < g - b$ and $c(e) < g - b$. By assuming that $c(e^{\max}) \leq (g - b)$, we know that these two conditions will be satisfied, since $c(e)$ is an increasing and convex function of e. Hence, the lemma.

4. Results

4.1 Under complete information

4.1.1 The Shapiro and Stiglitz model. The Shapiro and Stiglitz model considers the context in which both the firm and the worker are risk neutral and this is common knowledge to all the players. In our reformulation of this model as a sequential game, the problem of incentives is introduced by assuming that $c(e^{\max}) \leq (g - b)$. In order to eliminate shirking, the firm will offer an efficiency wage, higher than that required by the participation constraint. For an effort e, the efficiency wage is defined as the minimum wage at which there is no shirking given that the worker has accepted the contract. It is obtained as the solution to the following equation, where the left-hand side indicates the utility to the worker when he respects the contract, and the right-hand side gives the utility obtained by the worker when he shirks on the contract:

$$w - c(e) = s(g - c(e_0)) + (1-s)(w - c(e_0)).$$

Then the set of efficiency wage contracts $C_{f,i}$ can be defined as (e, w_i), where:

$$C_{f,i} = (e, w_i), \quad \text{with } w_i = g + \frac{1}{s}(c(e) - c(e_0)). \qquad [9]$$

At the efficiency wage w_i, the profit of the firm Π_f is given as follows:

$$\Pi_f(e, w_i) = qe - w_i = qe - g - \frac{1}{s}[c(e) - c(e_0)].$$

Taking the derivative of Π_f, with respect to effort e, we get the optimal effort e_i as:

$$e_i = c'^{-1}(sq).$$

Substituting the value e_i in equation [9], the optimal efficiency wage w_i is then obtained as:

$$w_i = g + \frac{1}{s}[c(e_i) - c(e_0)].$$

Recall that, by assumption, $c(e^{\max}) \leq (g - b)$. Since $c(e^{\max}) > 0$, this means that $g > b$, i.e. even if a worker is caught shirking, his compensation is higher than the unemployment benefit. Thus, $w_i > b + c(e_i)$, and therefore the worker will always accept the efficiency wage contract.[4]

The Nash equilibrium of this game is found by the standard method of backward induction. A worker respects only the efficiency wage contract. Since the profit of the firm is greater if the worker respects the contract than if he shirks, the firm prefers to offer the efficiency wage contract rather than the one satisfying only the participation constraint. Thus, the Nash equilibrium of the above game consists of the firm offering the efficiency wage contract to the worker and the worker respecting the same contract.

4.1.2 The efficiency wage contract with varying degrees of risk aversion. Let us now consider the general case in which the degree of risk aversion of the firm is given by $\delta_i = \delta$ and the degree of risk aversion of the worker is given by $\beta_i = \beta$. Under complete information the values of δ and β are common knowledge and we can examine how the terms of the efficiency wage contract vary as a function of δ and β.

Proposition 1: The efficiency wage contract as a function of the degree of risk aversion.

(i) Under complete information, both the terms (effort and wage) of the efficiency wage contract change with the degree of risk aversion of only the worker.

(ii) For any effort e, the higher the degree of risk aversion of the worker β the lower the corresponding efficiency wage $w_i(\beta)$.

Proof. (i) Recall that for an effort e, the efficiency wage is defined as the minimum wage at which there is no shirking, given that the worker has accepted the contract, i.e. it is the solution to the following equation:

$$(w-c(e))^{1-\beta} = s(g-c(e_0))^{1-\beta} + (1-s)(w-c(e_0))^{1-\beta}. \qquad [10]$$

Clearly, the efficiency wage is not only a function of effort but also the degree of risk aversion of the worker and therefore can be written as $w_i(\beta)$.

Now, given the degree of risk aversion of the worker, β, and the corresponding efficiency wage $w_i(\beta)$, the firm decides on a value of effort so as to maximize its utility $U_f(e, w_i(\beta))$. Recall that by definition the utility function of a firm is a monotonic transformation of its profit function, i.e. $U_f(e, w_i(\beta)) = [\Pi_f(e, w_i(\beta))]^{1-\delta}$. Thus, for a given degree of risk aversion of the worker, β, the solution to the following profit-maximization problem of the firm is identical for all firms, or:

$$\underset{e}{Argmax}[\Pi_f(e, w_i(\beta))]^{1-\delta} = \underset{e}{Argmax}[\Pi_f(e, w_i(\beta))]^{1-\hat{\delta}} \text{ for any } \hat{\delta} \neq \delta.$$

This means that the optimal effort corresponding to the efficiency wage contract is dependent only on the degree of risk aversion of the worker β. Given the above, the efficiency wage contract can be written as $C_f(\beta) = (e_i(\beta), w_i(\beta))$. It depends only on the degree of risk aversion of the worker and not on the degree of risk aversion of the firm.

(ii) In order to simplify the proof, we first formalize the notion of a lottery and introduce the concept of a certainty equivalent.

Consider the efficiency wage contract $(\bar{e}, \bar{w})$ formulated for an effort $\bar{e}$ and for a worker with a degree of risk aversion β. The certain payoff to the worker from respecting the efficiency wage contract is $(\bar{w} - c(\bar{e}))^{1-\beta}$. Let the lottery associated with shirking on this contract be given by $\bar{L} = (\bar{w} - c(e_0), g - c(e_0); (1 - s), s)$. This implies that the worker gets $(\bar{w} - c(e_0))$ with a probability $(1 - s)$ and $(g - c(e_0))$ with a probability s. Let the certainty equivalent of a lottery L be given by $CE(L)$. By definition, the certainty equivalent of a lottery L is a sum which yields the same utility as the expected utility of the lottery L or $U(CE(L)) = EU(L)$. This means that $(\overline{w} - c(\overline{e}))$ is the certainty equivalent of the lottery $\overline{L}$, since we know that:

$$(\overline{w} - c(\overline{e}))^{1-\beta} = s(g - c(e_0))^{1-\beta} + (1-s)(\overline{w} - c(e_0))^{1-\beta} = EU(\overline{L}).$$

Now let us further suppose that the efficiency wage contract $(\overline{e}, \overline{w})$ is formulated for a risk-neutral worker (i.e. $\beta = 0$). Let us consider the impact of this contract on a risk-loving worker with a degree of risk aversion, β_{RL}. It is a well-known result in microeconomics that the certainty equivalent of any lottery, including $\overline{L}$, is greater for a risk-loving agent than for a risk-neutral agent. This means that for a $\beta_{RL} < 0$ there exists an amount $x > \overline{w} > 0$ such that we have:

$$(x - c(\overline{e}))^{1-\beta_{RL}} = s(g - c(e_0))^{1-\beta_{RL}} + (1-s)(\overline{w} - c(e_0))^{1-\beta_{RL}}. \qquad [11]$$

Furthermore, for an effort $\overline{e}$, the efficiency wage constructed for a risk-loving worker, say w^{RL} would satisfy the following equation:

$$\left(w^{RL} - c(\overline{e})\right)^{1-\beta_{RL}} = s(g - c(e_0))^{1-\beta_{RL}} + (1-s)\left(w^{RL} - c(e_0)\right)^{1-\beta_{RL}}.$$

Substituting for the value of $(g - c(e_0))^{1-\beta_{RL}}$ from equation [11] in the above, keeping in mind that $x > \overline{w}$ and rearranging the terms, we get:

$$\begin{aligned}&\left(W^{RL} - c(\overline{e})\right)^{1-\beta_{RL}} - (1-s)\left(w^{RL} - c(e_0)\right)^{1-\beta_{RL}} \\ &\quad > (\overline{w} - c(\overline{e}))^{1-\beta_{RL}} - (1-s)(\overline{w} - c(e_0))^{1-\beta_{RL}}.\end{aligned}$$

Then, by simple examination of the terms on both sides, we can see that $w^{RL} > \overline{w}$, or for any given level of effort the efficiency wage for a risk-loving worker is greater than that for a risk-neutral worker. For other cases, i.e. between a risk averse and a risk neutral worker etc. it can be proved similarly that, for a given effort, the higher the degree of risk aversion the lower the efficiency wage.

The intuition behind the first result of proposition 1 is that the firm can set the efficiency wage so as to eliminate shirking on the part of the worker with probability one and, consequently, the firm does not face any uncertainty. The second result of proposition 1 reflects the fact that risk-averse workers need lower incentives to respect the contract and therefore they can be paid a lower premium for supplying the same effort as other types of workers.

This completes the proof of proposition 1.

Under complete information, the firm can clearly recognize the type of worker. Then it can offer the worker the efficiency wage contract corresponding to his degree of risk aversion. Since the efficiency wage contract is that which maximizes the profit of the firm, the Nash equilibrium of the game is still one in which the firm offers the efficiency wage contract corresponding to the degree of risk aversion of the worker (whatever the type of the firm) and the worker accepts and respects the efficiency wage contract. Then proposition 1 indicates that if the efficiency wage contract $C_f(\beta^*) = (e_i(\beta^*), w_i(\beta^*))$ is the Nash equilibrium contract when $\delta = \delta^*$ and $\beta = \beta^*$, then it will also be the Nash equilibrium contract for any other $\delta \neq \delta^*$ and $\beta = \beta^*$.

While we know that the efficiency wage contract will be offered at equilibrium, it is impossible to derive the terms of the efficiency wage contract analytically — especially the effort — as a function of the degree of risk aversion of the worker. Therefore, it is necessary to resort to numerical solutions.

4.1.3 Illustrations with numerical simulations.[5] The parameter configurations considered for the numerical simulations[6] are as follows: $g = 15$, $b = 12$, $s = 0.5$, $e_0 = 5.01$, $e^{\max} = 15$ and $q = 3.7$, 4.7 and 5.5. The degrees of risk aversion of the risk-averse worker, a risk-neutral worker and a risk-loving worker are, respectively, $\beta_{RA} = 0.5$, $\beta_{RN} = 0$ and $\beta_{RL} = -0.5$. The cost function $c(e)$ is convex in effort e and is given by $c(e) = 0.09e^2 - 2.25$.

The efficiency wage contract $(e_i(\beta), w_i(\beta))$ is computed as follows. A value of q and β is considered for each simulation. For each possible value of effort e, the efficiency wage $w_i(\beta)$ is computed using equation [10]. Once we have this schedule of $w_i(\beta)$, the optimal effort $e_i(\beta)$ is identified as the effort that maximizes $\Pi_f(e, w_i)$.

Simulation result 1. Comparative static analysis under complete information. For the given configuration of parameters, the terms of the efficiency wage contract are an increasing function of the degree of risk aversion of the worker, i.e.:

$$e_i(\beta_{RA}) > e_i(\beta_{RN}) > e_i(\beta_{RL});$$

$$w_i(e_i, \beta_{RA}) > w_i(e_i, \beta_{RN}) > w_i(e_i, \beta_{RL}).$$

Discussion. The above result can be clearly inferred from Table 1. The intuition may be given as follows. According to proposition

Table 1. Terms of the efficiency wage contract and profit of the firm

	q = 5.5		q = 4.7		q = 3.7	
	e_i	w_i	e_i	w_i	e_i	w_i
The efficiency wage contract						
with $\beta = 0.5$ (risk averse)	19.25	67.72	16.00	51.19	12.25	35.36
with $\beta = 0$ (risk neutral)	15.25	52.36	13.00	40.92	10.25	29.41
with $\beta = -0.5$ (risk loving)	12.50	41.79	11.00	34.15	9.00	25.79

Note: The values of the parameters are as follows: $g = 15$, $b = 12$, $s = 0.5$, $e_0 = 5.01$, $e^{\max} = 15$; and the cost function is: $c(e) = 0.09e^2 - 2.25$.

1(ii), for a given productivity of worker's effort, q, for *the same effort*, a risk-loving worker has to be paid more than a risk-averse worker to eliminate shirking. Turning the argument the other way round, this also means that for the same amount of investment on elimination of shirking a firm can demand a higher effort from a risk-averse worker than from a risk-loving worker. Again, since the effort demanded from a risk-averse worker is more, the wage paid to him is more.

From simulation result 1, we can see how the terms of the contract vary with the degree of risk aversion of the worker. When a worker is risk averse, the terms of the contract initiated are higher than those corresponding to a risk-neutral worker. Similarly, when the worker is risk loving, the terms of the contract initiated are lower than those corresponding to a risk-neutral worker.

4.2 Under incomplete information

We now introduce incomplete information for the firm with respect to the degree of risk aversion of the worker. We do not need to introduce incomplete information for the worker regarding the degree of risk aversion of the firm, since the latter has no impact on the response of the worker to any contract that he is offered. Then we have the following proposition explaining how shirking can occur.

Proposition 2. For any efficiency wage contract, a worker will shirk if the degree of risk aversion of the worker is less than that corresponding to the contract.

Proof. By definition, an efficiency wage contract is such that the utility from respecting the contract is equal to the expected utility from shirking on the contract.

Let (e, w) be an efficiency wage contract formulated by the firm for a risk-neutral worker. By definition, we have:

$$w - c(e) = s(g - c(e_0)) + (1 - s)(w - c(e_0)).$$

$$\Leftrightarrow (w - c(e))^{1-\beta_{RL}} = (s(g - c(e_0)) + (1 - s)(w - c(e_0)))^{1-\beta_{RL}}. \qquad [12]$$

Let $1 - \beta_{RL}$, where $\beta_{RL} < 0$, be the degree of risk aversion of a risk-loving worker. Since the utility function of a risk-loving worker is strictly convex, from the definition of a strictly convex function, we have:

$$(s(g - c(e_0)) + (1 - s)(w - c(e_0))^{1-\beta_{RL}}$$
$$< s(g - c(e_0))^{1-\beta_{RL}} + (1 - s)(w - c(e_0))^{1-\beta_{RL}}. \qquad [13]$$

Combining equations [12] and [13] we have:

$$(w - c(e))^{1-\beta_{RL}} < s(g - c(e_0))^{1-\beta_{RL}} + (1 - s)(w - c(e_0))^{1-\beta_{RL}}. \qquad [14]$$

The left-hand side of the above inequality [14] represents the utility to a risk-loving worker from respecting the efficiency wage contract formulated for a risk-neutral worker. The right-hand side represents the utility to a risk-loving worker from shirking on the efficiency wage contract formulated for a risk-neutral worker. Evidently, in such a context, the risk-loving worker will choose to shirk.

By the same type of reasoning it can be shown that when the above efficiency wage contract is offered to a risk-averse worker he will always respect it, keeping in mind that the utility function of a risk-averse worker is strictly concave and therefore inequality [14] will be in the opposite direction. The same method can be followed to show that if the efficiency wage contract is formulated for a risk-averse worker, it will be cheated upon by a risk-neutral or risk-loving worker. This completes our proof.

Let us now examine whether the efficiency wage contracts can serve as an effective screening device. For simplicity, we consider a discrete distribution with a specific degree of risk aversion β and δ for each type of worker and firm, respectively. Let β_{RA}, β_{RN} and β_{RL} be the degrees of risk aversion of the risk-averse, risk-neutral and risk-loving worker, respectively. Similarly for the firm (i.e. δ_{RA}, δ_{RN} and δ_{RL} are the degrees of risk aversion of the three types of firms). Let $C_{RA} = (e_{RA}, w_{RA})$; $C_{RN} = (e_{RN}, w_{RN})$ and $C_{RL} = (e_{RL}, w_{RL})$ be the efficiency wage contracts corresponding to a risk-averse, risk-neutral and a risk-loving worker, respectively. Then we have the following simulation result.

Simulation result 2. If the firm offers a set of efficiency wage contracts corresponding to the possible degrees of risk aversion of the workers, then it will not act as an effective screening device.

Discussion. Table 2 presents the results of the simulation on the payoffs to workers under the different efficiency wage contracts. Given the result of proposition 2, if the firm offers the contracts C_{RA}, C_{RN} and C_{RL}, all workers will choose the contract C_{RA} and only risk-averse workers will respect the contract.

Finally, we turn to the Bayesian-Nash equilibrium of the game under incomplete information. By definition, the Bayesian-Nash equilibrium of the above game consists of a set of strategies for each type of worker and each type of firm, which maximizes the expected payoff of each type of player, given the beliefs and the strategies pursued by the other players.[7] Let the beliefs of the firms that a worker is risk averse, risk neutral or risk loving be given by p_{RA}, p_{RN} and p_{RL}, respectively. This is assumed to be common knowledge. Let C_{RA}, C_{RN} and C_{RL} be the efficiency wage contracts corresponding to a risk-averse, risk-neutral and risk-loving worker, respectively. Lastly, let e_{RA}, e_{RN} and e_{RL} be the efficiency wage contract effort levels corresponding to the three types of efficiency wage contracts.

Table 2. Payoff to the worker from the different efficiency wage contracts

	Type of contract					
	$C_{RA} = (12.25, 35.36)$		$C_{RN} = (10.25, 29.41)$		$C_{RL} = (9.00, 25.79)$	
Type of worker	**Respect**	**Shirk**	**Respect**	**Shirk**	**Respect**	**Shirk**
Risk averse	4.91	4.91	4.71	4.64	4.55	4.47
Risk neutral	24.10	25.18	22.20	22.20	20.75	20.39
Risk loving	118.34	134.18	104.63	108.79	94.53	94.53

Note: The values of the parameters are as follows: $g = 15$, $b = 12$, $s = 0.5$, $e_0 = 5.01$, $e^{\max} = 15$, $q = 3.7$ and the cost function is $c(e) = 0.09e^2 - 2.25$.

From proposition 2, we know that whenever the contract C_{RL} is offered, all types of workers will respect it. However, if the contract C_{RA} is given, only a risk-averse worker will respect it, and finally if C_{RN} is offered, a risk-averse or a risk-neutral worker will respect it. This means that the returns to a firm from the contract C_{RL} is certain, but the expected returns from the efficiency wage contracts C_{RA} and C_{RN} constitute lotteries (actually compound lotteries) for a firm, as given below:

$$E\Pi_f(C_{RA}) = p_{RA} \cdot (\Pi_f(C_{RA}, e_{RA}))^{1-\delta} + (1 - p_{RA}) \cdot (\Pi_f(C_{RA}, e_0))^{1-\delta} \quad [15]$$

$$E\Pi_f(C_{RN}) = (1 - p_{RL}) \cdot (\Pi_f(C_{RN}, e_{RN}))^{1-\delta} + p_{RL} \cdot (\Pi_f(C_{RN}, e_0))^{1-\delta} \quad [16]$$

$$E\Pi_f(C_{RL}) = (p_{RA} + p_{RN} + p_{RL}) \cdot (\Pi_f(C_{RL}, e_{RL}))^{1-\delta}. \quad [17]$$

Given the beliefs p_{RA}, p_{RN} and p_{RL}, a firm compares the expected returns from the lotteries associated with the contracts C_{RA} and C_{RN}, and the certain payoff associated with the contract C_{RL}. It then selects the contract that yields the maximum payoff. Suppose nature picks a risk-averse worker at the beginning of the game, then whatever the type of firm, under the above belief structure shirking will never be observed at equilibrium, according to our proposition. However, if nature picks a risk-neutral worker or a risk-loving worker, then shirking will be observed whenever the firm offers the contracts C_{RA} or C_{RN}. The above arguments, along with the propositions and simulations presented so far, lead to the following corollary presenting the properties of the Bayesian-Nash equilibrium strategies.

Corollary.

(i) The degree of risk aversion of the firm does not influence the equilibrium contract under complete information, but it does determine the equilibrium contract under incomplete information.

(ii) Symmetric firms will play symmetric pure strategies at equilibrium, i.e. offer the same type of contract at equilibrium under both complete and incomplete information.

(iii) If the firms are not symmetric, i.e. if they have different beliefs or different degrees of risk aversion, there can be a dispersion of contracts and wages in the market under incomplete information.

(iv) Under incomplete information, the probability of shirking at equilibrium will depend on the proportion of risk-averse, risk-neutral or risk-loving firms as well as workers in the market.

Proof.

(i) Proposition 1 shows that under complete information, the terms of the efficiency wage contract depend only on the degree of risk aversion of the worker. Furthermore, under complete information, the firm knows the type of worker. Therefore, at equilibrium, the firm offers the efficiency wage contract designed for the type of worker independently of its own degree of risk aversion. Under incomplete information, for any set of beliefs of the firms about the nature of the workers p_{RA}, p_{RN} and p_{RL}, the Bayesian-Nash equilibrium strategy of the firm is the contract that maximizes its expected utility. According to equations [15]–[17] this in turn depends on the firm's degree of risk aversion.

(ii) Under complete information, all firms recognize the degree of risk aversion of the worker and offer the corresponding wage contract. Under incomplete information, if all firms have the same *beliefs* p_{RA}, p_{RN} and p_{RL}, and the same *degree of risk aversion*, δ, then according to equations [15]–[17] the same type of contract will yield the maximum payoff for all of them. Therefore, under both informational contexts, symmetric firms will offer the same type of contract at equilibrium.

(iii) This follows directly from the above two points (i) and (ii).

(iv) Shirking emerges as an equilibrium whenever the contract C_{RA} is offered to a risk-neutral or risk-loving worker, or when the contract C_{RN} is offered to a risk-loving worker. Under incomplete information, the probability of a firm offering C_{RA}, C_{RN} and C_{RL} depends on the firm's degree of risk aversion and the proportion of the different types of workers in the market. This means that the probability of contracts C_{RA} or C_{RN} being offered in a market with subsequent shirking by workers will depend both on the ratios of the different types of workers and the ratios of the different types of firms in the market.

5. Conclusion

The main objective of the present paper was to examine the impact of a firm's and worker's degree of risk aversion on the formulation of the efficiency wage contract under complete and incomplete information. Towards this aim, the Shapiro–Stiglitz (1984) model was reformulated as a game with incomplete information about the degree of risk aversion of the worker. The paper showed that the terms of the efficiency wage contract depended only on the worker's degree of risk aversion. Simulations indicated that for the given parameter configurations, the terms of the efficiency wage contract were an increasing function of the worker's degree of risk aversion. It also explained that under incomplete information, shirking would be observed at equilibrium whenever a worker accepted a contract such that the degree of risk aversion corresponding to the contract was more than his actual degree of risk aversion.

It can be argued that it is not surprising to observe shirking as an equilibrium outcome, since we have introduced incomplete information with no possibility for 'screening'. While this is true, we wish to reiterate that by not allowing for the elimination of incomplete information through screening we are forced to examine how beliefs on player types influence contract formulation. Furthermore, by considering heterogeneity in terms of players' degree of risk aversion, we are able to study the relation between the degree of risk aversion, firm beliefs and contract formulation by a firm.

In addition to the above, the present model yields a few insights on some other debates in labour economics. For instance, there is an ongoing debate on the differentiation of wages in the same industry for similarly qualified workers. Fehr *et al.* (1996) and Krueger and Summers (1988) propose that wage differentials (given two identical workers) could be due to different firm-specific production technologies. In this case, firms with a more efficient production technology would offer a higher salary.

Our model suggests that, under complete information, wage differentials could also occur due to workers being differentiated according to their degree of risk aversion. Suppose that employment in a sector is generated by the repetition of the game specified in our model (with one firm and one worker playing at a time). Under complete information, a firm can offer different efficiency wages to different workers according to their degree of risk aversion. Already this creates a wage differential on the basis of the workers' degree of risk aversion.

Under incomplete information, in each round of the game, the firm starts out with an a priori belief on the degree of risk aversion of the worker. In the present paper, for simplicity, we assume that all beliefs are common knowledge. In this case, wage differentials emerge as a result of variations in the firms' degree of risk aversion. On the other hand, in emerging sectors, such as new knowledgeintensive high-tech industries, firms might have subjective rather than objective beliefs. Then, each firm can offer different efficiency wage contracts given its firm-specific beliefs. Thus, in generalizing the Shapiro–Stiglitz model to incorporate varying degrees of risk aversion, the present paper shows that firms with the same production technology can initiate different contracts given *workers* with different degrees of risk aversion (under complete information) or given *firms* with different degrees of risk aversion or different beliefs (under incomplete information).

Notes

[1] Social custom models also exist that examine the context where the firm knows the proportion of different types of workers and formulates its efficiency wage contract accordingly (Chang and Lai, 1999).

[2] Essentially in such cases, the net revenue generated by a contract after covering the costs of screening is less than the expected payoff from an efficiency wage contract that does not screen, i.e. an efficiency wage contract that eliminates opportunism with a probability strictly less than 1. In our model this occurs under the assumption that the worker's marginal productivity of effort is high enough to sustain an efficiency wage contract, but not high enough to sustain an efficiency wage contract and the costs of screening.

[3] δ and β cannot be greater than one, for otherwise the first derivative or the marginal utility of payoffs will be negative. Furthermore, note that the Arrow–Pratt measure of absolute risk aversion is given by $r(.) = -U''(.)/U'(.)$. Since the utility function of the worker is given by $U(\Pi_t) = \Pi_t^{1-\beta}$, the Arrow–Pratt measure of absolute risk aversion for the worker is given by:

$$r(\Pi_l) = -\frac{(-\beta)(1-\beta)(\Pi_l)^{-\beta-1}}{(1-\beta)(\Pi_l)^{-\beta}} = \beta(\Pi_l)^{-1}.$$

Clearly, $r > 0$ if $\beta > 0$; $r = 0$ if $\beta = 0$ and $r < 0$ if $\beta < 0$. Similarly for the firm.

[4] Given the definition of the efficiency w_i and the fact that $s < 1$, we have: $w_i = g + 1/s\ (c(e_i) - c(e_0)) > g + (c(e_i) - c(e_0))$. Furthermore, since by assumption $g > c(e^{\max}) + b$, we can write $w_i > b + c(e^{\max}) + c(e_i) - c(e_0)$. Finally, as $c(e^{\max}) - c(e_0) > 0$, we have $w_i > b + c(e_i)$.

[5] Simulations were conducted using the Mathcad program.

[6] The parameter configurations considered for the numerical simulations are the same as those used in the experiments of Fehr *et al.* (1996).

[7] The equilibrium concept used in our game is the Bayesian-Nash equilibrium rather than the perfect Bayesian-Nash equilibrium because there is no updating of beliefs. In our game, the firm makes the first move, offering a contract based on its own beliefs. Thereafter, the worker responds according to his type, and his beliefs about the type of firm have no influence on his action. Therefore, the updating of beliefs by the worker has no impact on the equilibrium strategies and hence need not be considered.

References

Akerlof G. A. and Yellen J. L. (1986) *Efficiency Wage Models of the Labor Market*, Cambridge: Cambridge University Press.

Albrecht J. W. and Vroman S. B. (1998) 'Nash Equilibrium Efficiency Wage Distributions', *International Economic Review* 39: 183–203.

Bowles S. (1985) 'The Production Process in a Competitive Economy: Walrasian, Neo-Hobbesian, and Marxian Models', *American Economic Review* 75: 16–36.

Cappelli P. and Chauvin K. (1991) 'An Interplant Test of the Efficiency Wage Hypothesis', *Quarterly Journal of Economics* 106: 769–787.

Chang J. and Lai C. (1999) 'Carrots or Sticks? A Social Custom Viewpoint on Worker Effort', *European Journal of Political Economy* 15: 297–310.

Fehr E., Kirchsteiger R. and Riedl A. (1996) 'Involuntary Unemployment and Non-compensating Wage Differentials in an Experimental Labour Market', *The Economic Journal* 106: 106–121.

Katz L. F. (1986) 'Efficiency Wage Theories: A Partial Evaluation', *NBER Macroeconomics Annual*: 235–289.

Krueger A. B. and Summers L. H. (1988) 'Efficiency Wage and the Inter-industry Wage Structure', *Econometrica* 56: 259–293.

Mas-Collel A., Whinston M. D. and Green J. R. (1995) *Microeconomic Theory*, Oxford: Oxford University Press.

Shapiro C. and Stiglitz J. E. (1984) 'Equilibrium Unemployment as a Worker Discipline Device', *American Economic Review* 74: 433–444.

Sparks R. (1986) 'A Model of Involuntary Unemployment and Wage Rigidity: Worker Incentives and the Threat of Dismissal', *Journal of Labor Economics* 4: 560–581.

Strand J. (1987) 'Unemployment as a Discipline Device with Heterogeneous Labor', *American Economic Review* 77: 489–493.

Weiss A. (1991) *Efficiency Wages Models of Unemployment, Layoffs and Wage Dispersion*, Oxford: Clarendon Press.

Yellen J. (1984) 'Efficiency Wage Models of Unemployment', *American Economic Review* 74: 200–205.

Zenou Y. (1996) 'La Théorie du salaire d'efficience: de nouveaux fondements microéconomiques dans la détermination des salaires et du chômage' in Ballot G. (ed.), *Les Marchés internes du travail: de la micro à la micro*, Paris: Presses Universitaires de France.